Jinan University

Faculty of Business Administration
Tripoli - Lebanon

General Mathematics

Preparatory Entrance Exam

Version 1

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Jinan University
Faculty of Business Administrations

Contents

1 Functions, Limits, and Continuity

THE STUDY OF A REAL FUNCTION

1. **Find the domain of definition D_f of the function f (when it is not given).**

2. **Study the limits of f.** We study the limits of the function f at the end points of the domain of definition D_f, and deduce the asymptotes of f.

   (a) **Horizontal asymptote:** if $\lim\limits_{x \to +\infty} f(x) = a$ (respectively $\lim\limits_{x \to -\infty} f(x) = a$), then the line of equation $y = a$ is a horizontal asymptote of f at $+\infty$ (respectively at $-\infty$).

   (b) **Verticale asymptote:** if $\lim\limits_{x \to a} f(x) = +\infty$ (respectively $\lim\limits_{x \to a} f(x) = -\infty$), then the line of equation $x = a$ is a vertical asymptote of f at $+\infty$ (respectively at $-\infty$).

   (c) **Oblique asymptote:** Let (D) be the line of equation $y = ax + b$. If $\lim\limits_{x \to +\infty} [f(x) - (ax + b)] = 0$ (respectively $\lim\limits_{x \to -\infty} [f(x) - (ax + b)] = 0$), then (D) is an oblique asymptote of f at $+\infty$ (respectively at $-\infty$).

3. **Calculate the first derivative of f**

4. **Draw the table of variation of f**

5. **Contruct the asymptotes and the representative curve (C_f) of** f, using the particular points.

Exercise 1.1 *Study the limits of f at the end points of its .*

1. $f(x) = x^3 - 2x^2 - 4x - 1$.

Answer: $\pm\infty$.

2. $f(x) = \dfrac{x^2 - x + 5}{2x - 3}$.

   Answer: $\pm\infty$.

3. $f(x) = \dfrac{x^2 + 2x + 2}{x^2 - 3x - 4}$.

   Answer: $1, \pm\infty$.

4. $f(x) = -x + 1 + \dfrac{1}{x + 2}$.

   Answer: $\mp\infty$.

Exercise 1.2 *Let f be a function defined on $]0, +\infty[$ by $f(x) = \sqrt{x+1} - \sqrt{x}$.*

1. *Show that $0 \le f(x) \le \dfrac{1}{2\sqrt{x}}$ for all $x > 0$.*

2. *Deduce the limit of f at $+\infty$.*

   Answer: 0.

Exercise 1.3 *Find the following limits:*

1. $\displaystyle\lim_{x \to +\infty} \sqrt{\dfrac{4x + 1}{x - 1}}$.

   Answer: 2.

2. $\displaystyle\lim_{x \to +\infty} \sqrt{\dfrac{x^2 - 1}{4x^2}}$.

   Answer: 1/2.

2 Derivatives

In the 19th century, economists developed the concept of marginal analysis, such as marginal cost,marginal product and others...

If $C(x)$ is the total cost of manufacturing x units of a certain item, once x items have been produced, $C'(x)$ is the marginal cost.

Standard Formula:

$f(x)$	$f'(x)$
C	0
x^n	nx^{n-1}
e^x	e^x
$ln(x)$	$\dfrac{1}{x}$
$\dfrac{1}{x}$	$-\dfrac{1}{x^2}$
$\sqrt{x}$	$-\dfrac{1}{2\sqrt{x}}$

Exercise 2.1 *Find the derivative of each of the following functions f:*

1. $f(x) = \dfrac{3x+1}{x-3}$, $x \in]-\infty, 3[\cup]3, +\infty[.$

   Answer: $\dfrac{-10}{(x-3)^2}.$

2. $f(x) = \dfrac{2x + 3}{x^4 + 1}$.

   Answer: $\dfrac{-6x^4 - 12x^3 + 2}{(x^4 + 1)^2}$

Exercise 2.2 *Let I be an interval of $\mathbb{R}$, and let $u : I \longrightarrow \mathbb{R}$ is dfferentiable in I. Find the derivative of each of the following functions:*

1. $[u(x)]^n$ $(n \in \mathbb{N}^*)$.

   Answer: $n[u(x)]^{n-1}u'(x)$.

2. $e^{u(x)}$.

   Answer: $e^{u(x)}u'(x)$.

3. $\sqrt{u(x)}$ *(on suppose que $u(x) > 0$)*.

   Answer: $\dfrac{u'(x)}{2\sqrt{u(x)}}$.

4. $\ln|u(x)|$ *(on suppose que $u(x) \neq 0$ pour tout $x \in I$)*.

   Answer: $\dfrac{u'(x)}{u(x)}$.

3 Integrals

Standard Formula:

$\int f(x)dx$	$F(x)$
$\int x^n dx$	$\dfrac{x^{n+1}}{n+1} + c$
$\int e^x dx$	$e^x + c$
$\int \dfrac{1}{x} dx$	$ln|x| + c$
$\int \dfrac{1}{x^2} dx$	$-\dfrac{1}{x}$
$\int \sqrt{x} dx$	$-\dfrac{2}{3} x\sqrt{x} + c$

Exercise 3.1 *Calculate the following integrals:*

1. $\displaystyle\int (4x^3 - 2x + 1)dx \ x \in]0, +\infty[.$

   Answer: $x^4 - x^2 + x + c.$

2. $\displaystyle\int (\dfrac{2}{x} + e^x)dx. \ x \in] -\infty, 0[\cup]0, +\infty[.$

   Answer: $2ln|x| + e^x + c.$

Exercise 3.2 *Calculate the following definite integrals:*

1. $\displaystyle\int_1^e lnx dx.$

   Answer: 1.

2. $\displaystyle\int_0^1 x^2 e^x dx.$

   Answer: $e - 2.$

3. $\displaystyle\int_0^1 \frac{1}{(x+1)(x+2)} dx$

   Answer: *2ln2-ln3.*

4 Differential Equations

An equation involving an unknown function and one or more of its derivatives is called a differential equation. The order of a differential equation is the order of the highest derivative that appear in it.

Exercise 4.1 *Solve the following differential equations:*

1. $x' - x = 0$

   Answer: $x = ke^t$.

2. $tx' - x = 0$

   Answer: $x = kt$.

Exercise 4.2 *Solve the following differential equations:*

1. $x'' - 3x' + 2x = 0$

   Answer: $x = C_1 e^t + C_2 e^{2t}$.

2. $x'' + 4x' - 5x = 0$

   Answer: $x = C_1 e^t + C_2 e^{-5t}$.

3. $x'' + 7x' + 6x = 0$

   Answer: $x = C_1 e^{-t} + C_2 e^{-6t}$.